CSCI 1900
Discrete Structures

Sets and Subsets
Reading: Kolman, Section 1.1

Definitions of sets

- A ***set*** is any well-defined collection of objects
- The elements or members of a set are the objects contained in the set
- Well-defined means that it is possible to decide if a given object belongs to the collection or not.

Set notation

- Enumeration of sets are represented with a list of elements in curly brackets – example: {1, 2, 3}
- Set labels are uppercase letters in italics – example: *A*, *B*, *C*
- Element labels are lowercase letters – example: a, b, c
- ∅ is the label for the empty set, i.e., ∅ = { }

Membership

- $\in$ -- "is an element of" (Note that it is shaped like an "E" as in **e**lement)
- $\notin$ -- "is not an element of"
- Example: If A = {1, 3, 5, 7}, then 1 $\in$ A, but 2 $\notin$ A.

Specifying sets with their properties:

A set can be represented or defined by the rules classifying whether an object belongs to a collection or not.
A = {a | a is ___________}
The above notation translates to "the set A is comprised of elements a where a satisfies _________."

Examples of Common Sets

- Z^+ = {*x* | *x* is a positive integer}
- *N* = {*x* | *x* is a positive integer or zero}
- Z = {*x* | *x* is an integer}
- Q = {*x* | *x* is a rational number}
  Q consists of the numbers that can be written a/b, where a and b are integers and b ≠ 0.
- R = {*x* | *x* is a real number}

Examples of sets used in programming

- unsigned int
- int
- float
- enum

Subsets

- If every element of A is also an element of B, that is, if whenever $x \in A$, then $x \in B$, we say that A is a subset of B or that A is contained in B.
- A is a subset of B if for every x, $x \in A$ means that $x \in B$.

Subset Notation

- $\subseteq$ -- "is contained in" (Note that it is shaped like a "C" as in **c**ontained in)
  $\nsubseteq$ -- "is not contained in"
- "Is not contained in" does not mean that there aren't some elements that can be in both sets. It just means that not all of the elements of A are in B

Subset Examples

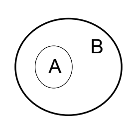
- vowels $\subseteq$ alphabet
- letters that spell "see" $\subseteq$ letters that spell "yes"
- letters that spell "yes" $\subseteq$ letters that spell "easy"
- letters that spell "say" $\subseteq$ letters that spell "easy"
- positive integers $\subseteq$ integers
- odd integers $\subseteq$ integers
- integers $\subseteq$ floating point values (real numbers)

Venn Diagrams

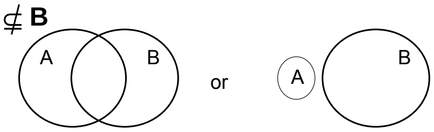
- Named after British logician John Venn
- Graphical depiction of the relationship of sets.
- Does not represent the individual elements of the sets, rather it implies their existence

Venn Diagram Examples

- **$A \subseteq B$**

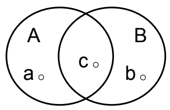


- **$A \nsubseteq B$**



More Venn Diagram Examples

a ∈ A, b ∈ B, and c ∈ A and c ∈ B

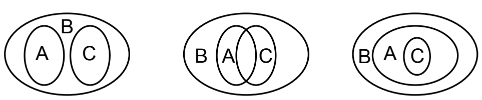


Theorems on Sets

- A ⊆ B and B ⊆ C implies A ⊆ C
- Example:
  - A = {*x* | letters that spell "see"} = {e, s}
  - B = {*x* | letters that spell "yes"} = {e, s, y}
  - C = {*x* | letters that spell "easy"} = {a, e, s, y}

Theorems on Sets (continued)

- If A ⊆ B and B ⊆ C, then A ⊆ C.
- If A ⊆ B and C ⊆ B, that doesn't mean we can say anything at all about the relationship between A and C. It could be any of the following three cases:



Theorems on Sets (continued)

- If A is any set, then A ⊆ A. That is, every set is a subset of itself.
- Since ∅ contains no elements, then every element of ∅ is contained in every set. Therefore, if A is any set, the statement ∅ ⊆ A is always true.
- If A ⊆ B and B ⊆ A, then A = B

Universal Set

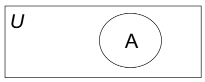
- There must be some all-encompassing group of elements from which the elements of each set are considered to be members of or not.
- For example, to create the set of integers, we must take elements from the universal set of all numbers.

Universal Set (continued)

- It's desirable for the all-encompassing set to make sense.
- Example: Although it is true that students enrolled in this class can be taken from the universal set of all mammals that roam the earth, it makes more sense for the universal set to be people enrolled at ETSU or at least limit the universal set to humans.

Universal Set (continued)

- The ***Universal Set U*** is the set containing all objects for which the discussion is meaningful. (e.g., examining whether a sock is an integer is a meaningless exercise.)
- Any set is a subset of the universal set from which it derives its meaning
- In Venn diagrams, the universal set is denoted with a rectangle.



Final set of terms

- ***Finite*** – A set A is called finite if it has n distinct elements, where $n \in N$.
- n is called the ***cardinality*** of A and is denoted by |A|, e.g., n = |A|
- ***Infinite*** – A set that is not finite is called infinite.
- ***The power set*** of A is the set of all subsets of A including $\varnothing$ and is denoted ***P(A)***